HW 5 2012

Saturday, August 18, 2012
11:26 PM

50

(a) $900\ddot{a}_{91} = 900 + 720\left(\frac{1}{1.04}\right) + 432\left(\frac{1}{1.04}\right)^2 + 216\left(\frac{1}{1.04}\right)^3$

$\ddot{a}_{91} = \boxed{2.42638}$

(b) $a_{91} = \ddot{a}_{91} - 1 = \boxed{1.42638}$

(c)

$900\ddot{a}_{91:\overline{3}|} = 900 + 720\left(\frac{1}{1.04}\right) + 432\left(\frac{1}{1.04}\right)^2$

$\ddot{a}_{91:\overline{3}|} = 1 + .8\frac{1}{1.04} + .48\left(\frac{1}{1.04}\right)^2$

$= \boxed{2.21302}$

(d) $Var[Y] = \dfrac{{}^2A_{91} - A_{91}}{d^2}$

$${}^2A_{91} = \frac{180v^2 + 288v^4 + 216v^6 + 216v^8}{900}$$

$= 0.82349$

$$A_{91} = \frac{180v + 288v^2 + 216v^3 + 216v^4}{900}$$

$= 0.90668$

$.82349 - (.90668)^2$

$$Var = \frac{.82349 - (.90668)^2}{\left(\frac{.04}{1.04}\right)^2}$$

$$= \boxed{0.96334}$$

(e)

$$\ddot{a}_{91:\overline{3}|} = 1 + v + v^2 + v^3\left(\frac{216}{900}\right)$$

$$= \boxed{3.09945}$$

(f) $$Var[Y] = \frac{{}^2A_{91:\overline{3}|} - (A_{91:\overline{3}|})^2}{d^2}$$

$${}^2A_{91:\overline{3}|} = \frac{180v^2 + 288v^4 + 432v^6}{900}$$

$$= 0.83780$$

$$A_{91:\overline{3}|} = \frac{180v + 288v^2 + 432v^3}{900}$$

$$= 0.91488$$

$$Var = \frac{0.83780 - (0.91488)^2}{\left(\frac{.04}{1.04}\right)^2}$$

$$= \boxed{0.53197}$$

(g) $\ddot{a}_{9} = E \; \ddot{a}_{9} =$

(g) $_{21}\ddot{a}_{91} = {}_{2}E_{91}\,\ddot{a}_{93} =$

$$v^2\left(\frac{432}{900}\right)\left(1+\frac{216}{432}v\right) = \boxed{0.65715}$$

(h) $$\bar{a}_{91} = \frac{1-\bar{A}_{91}}{\delta} = \frac{1-\frac{i}{\delta}A_{91}}{\delta}$$

$$= \frac{1-\frac{.04}{\ln(1.04)}(.90668)}{\ln(1.04)} \quad \text{Part (d)}$$

$$= 1.9200$$

(i) $$Var[Y] = \frac{{}^2\bar{A}_{91} - (\bar{A}_{91})^2}{\delta^2}$$

$$= \frac{\frac{[(1+i)^2-1]}{2\delta}\,{}^2A_{91} - \left(\frac{i}{\delta}A_{91}\right)^2}{\delta^2}$$

Part (d)

$$= \frac{\frac{[(1.04)^2-1]}{2\ln[1.04]}(.82349) - \left[\frac{.04}{\ln(1.04)}(0.90668)\right]^2}{[\ln(1.04)]^2}$$

$$= \boxed{1.03247}$$

$= \boxed{1.03247}$

(j) $(I\ddot{a})_{91} =$

$$\frac{900(1) + 720(2)v + 432(3)v^2 + 216(4)v^3}{900}$$

$= \boxed{4.72326}$

(k) $(I\ddot{a})_{91:\overline{3|}} =$

$$\frac{900(1) + 720(2)v + 432(3)v^2}{900}$$

$= \boxed{3.86982}$

(51)

(a) $\ddot{a}_{60} = \boxed{11.1454}$ ← straight from Table

(b) $a_{60} = \ddot{a}_{60} - 1 = \boxed{10.1454}$

(c) ${}_{10|}\ddot{a}_{60} = {}_{10}E_{60} \; \ddot{a}_{70}$

$= (0.45120)(8.5693)$

$= \boxed{3.86647}$

$= \boxed{3.86647}$

(d) $\ddot{a}_{60:\overline{20|}} = \ddot{a}_{60} - {}_{20}E_{60}\,\ddot{a}_{80}$

$= 11.1454 - (0.14906)(5.9050)$

$= \boxed{10.26520}$

(e) $\bar{a}_{80} = \dfrac{1-\bar{A}_{80}}{\delta} = \dfrac{1 - i/\delta\, A_{80}}{\delta}$

$= \dfrac{1-(1.02971)(0.66575)}{0.058827}$

$= \boxed{5.39628}$

(f) $\bar{a}_{50:\overline{20|}} = \dfrac{1-\bar{A}_{50:\overline{20|}}}{\delta}$

$= \dfrac{1-\left\{\bar{A}_{50} - {}_{20}E_{50}\,\bar{A}_{70} + {}_{20}E_{50}\right]}{\delta}$

$\dfrac{1-\left(\frac{i}{\delta}\right)(A_{50}) + {}_{20}E_{50}\left(\frac{i}{\delta}\right)A_{70} - {}_{20}E_{50}}{\delta}$

$= \dfrac{1-(1.02971)(.24905) + (.23047)(1.02971)(.51495) - 0.23047}{}$

$.05827$

$= \boxed{10.90248}$

(g) $\ddot{a}_{60:\overline{10}|} = \ddot{a}_{\overline{10}|} + {}_{10}E_{60}\,\ddot{a}_{70}$

$= \dfrac{1 - v_{.06}^{10}}{.06}(1.06) + (0.45120)(8.5693)$

$= \boxed{11.6682}$

(h) $\ddot{a}_{60:\overline{13}|} = \ddot{a}_{\overline{13}|} + {}_{13}E_{60}\,\ddot{a}_{73}$

$= \dfrac{1 - v_{.06}^{13}}{.06}(1.06) + v^{13}\dfrac{l_{73}}{l_{60}}\,\ddot{a}_{73}$

$= 9.3838 + \left(\dfrac{1}{1.06}\right)^{13}\left(\dfrac{5,920,394}{8,188,074}\right)(7.7568)$

$= \boxed{12.0134}$

(i) $\ddot{a}_{60}^{(12)} = \dfrac{1 - A_{60}^{(12)}}{d^{(12)}} = \dfrac{1 - \frac{i}{i^{(12)}} A_{60}}{d^{(12)}}$

Table

$= \dfrac{1 - (1.02721)(.36913)}{0.05813} = \boxed{10.6800}$

Table

(j) $\ddot{a}^{(12)}$ ...

(j) $\ddot{a}_{60}^{(12)} = \alpha(12)\,\ddot{a}_{60} - \beta(12)$

$= 1.00028(11.1454) - 0.46812$

$= \boxed{10.6804}$

(k) $\ddot{a}_{60}^{(12)} = \ddot{a}_{60} - \frac{12-1}{2(12)} - \frac{12^2-1}{12^3}(\delta+\mu_x)$

$$\boxed{\begin{aligned}\mu_x &\approx -\tfrac{1}{2}\left[\ln(p_{59}) + \ln(p_{60})\right]\\ &= -\tfrac{1}{2}\left[\ln(1-0.01262) + \ln(1-0.01376)\right]\\ &= 0.013278\end{aligned}}$$

$= 11.1454 - \frac{11}{24} - \frac{143}{1728}(0.05827 + 0.013278)$

$= \boxed{10.6811}$

(l) $\dfrac{{}^2A_{60} - (A_{60})^2}{d^2} = \dfrac{0.17741 - (0.36913)^2}{\left(\frac{.06}{1.06}\right)^2}$

$= \boxed{12.8443}$

(m) $\dfrac{{}^2A_{60:\overline{20|}} - (A_{60:\overline{20|}})^2}{}$

$$d^2$$

$${}^2A_{60:\overline{20|}} = {}^2A_{60} - v^{20}\,{}_{20}E_{60}\,{}^2A_{80} + v^{20}\,{}_{20}E_{60}$$

$$= 0.17741 - \left(\frac{1}{1.06}\right)^{20}(.14906)(0.47359) + \left(\frac{1}{1.06}\right)^{20}(.14906)$$

$$= 0.20188$$

$$A_{60:\overline{20|}} = A_{60} - {}_{20}E_{60}\,A_{80} + {}_{20}E_{60}$$

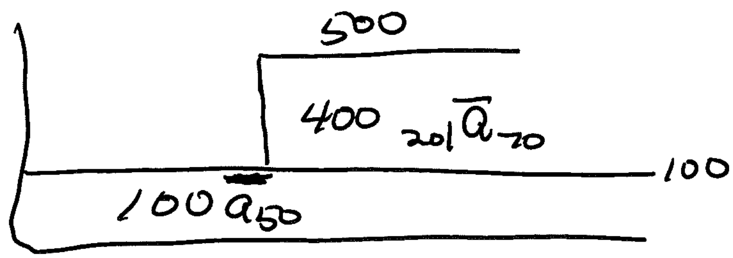
$$= 0.36913 - (.14906)(.66575) + 0.14906$$

$$= 0.41895$$

$$Var = \frac{0.20188 - (0.41895)^2}{\left(\frac{.06}{1.06}\right)^2}$$

$$= \boxed{8.22550}$$

52



$$100\,\frac{1-\bar{A}_{50}}{\delta} + 400\,{}_{20}E_{50}\,\frac{1-\bar{A}_{70}}{\delta}$$

$$= 100\,\frac{1-(1.02971)(.24905)}{}$$

$$.05827$$

$$+(400)(.23047)\ \frac{1-(1.02971)(.51495)}{.05827}$$

$$=\boxed{2019.23}$$

53 $(12)(800)\ \ddot{a}^{(12)}_{\overline{60:\overline{5|}}}$

$$(12)(800)\left[\frac{1-v^5}{d^{(12)}}+{}_5E_{60}\cdot\frac{1-\frac{1}{i^{(12)}}A_{65}}{d^{(12)}}\right]$$

$$=9600\left[4.34787+(.68756)\left(\frac{1-(1.02721)(.4398)}{.05813}\right)\right]$$

$$=\boxed{103,990.63}$$

54

ⓐ $\ddot{a}_{[54]:\overline{3|}}$

$$=1+v^1\,{}_1p_{[54]}+v^2\,{}_2p_{[54]}$$

$$=1+\frac{1}{1.06}\left[1-q_{[54]}\right]+$$

$$\left(\frac{1}{1.06}\right)^2\left[1-q_{[54]}\right]\left[1-q_{[54]+1}\right]$$

$$=1-\frac{.96}{1.06}+\frac{(.96)(0.946)}{(1.06)^2}$$

$$=\boxed{2.71307}$$

(b) $a_{[54]:\overline{3}|} = v\,{}_1p_{[54]} + v^2\,{}_2p_{[54]} + v^3\,{}_3p_{[54]}$

$= \frac{.96}{1.06} + \frac{(.96)(.945)}{(1.06)^2} + \frac{(.96)(.945)(.924)}{(1.06)^3}$

$= \boxed{2.41688}$

55 $\ddot{a}_{90} = 1 + v\,p_{90}\,\ddot{a}_{91}$

$= 1 + \frac{.85}{1.06}(3.4611)$

$= \boxed{3.77541}$

56 $\ddot{a}_{50:\overline{10}|} - 1 + {}_{10}E_{50} = a_{50:\overline{10}|}$

$8.2066 - 1 + v^{10}(.9195) = 7.8277$

$v^{10} = 0.675476$

$i = \left(\frac{1}{0.675476}\right)^{\frac{1}{10}} - 1$

$= \boxed{0.040014\%}$

57 $\ddot{a}_{60} = a_{60} + 1 = 11.996$

$\ddot{a}_{61} = a_{61} + 1 = 11.756$

$\ddot{a}_{62} = a_{62} + 1 = 11.509$

$\ddot{a}_{60} = 1 + v\, p_{60}\, \ddot{a}_{61}$

$11.996 = 1 + \left(\frac{1}{1.06}\right) p_{60} (11.756)$

$p_{60} = 0.991473$

$\ddot{a}_{61} = 1 + v\, p_{61}\, \ddot{a}_{62}$

$11.756 = 1 + \frac{1}{1.06} (p_{61})(11.509)$

$p_{61} = 0.990647$

$_2p_{60} = p_{60} \cdot p_{61} = (.991473)(.990647)$

$= \boxed{0.98220}$

57 a

$\ddot{a}_{[40]:\overline{4|}} = 1 + v\, {}_1p_x + v^2\, {}_2p_x + v^3\, {}_3p_x$

$= 1 + \frac{1}{1.06}\left(\frac{33485}{33519}\right) + \left(\frac{1}{1.06}\right)^2 \frac{33,440}{33,519}$

$+ \left(\frac{1}{1.06}\right)^3 \left(\frac{33,328}{33519}\right)$

$= 3.66643$

b

$$a_{[40]+1:\overline{4|}} = v\,{}_1p_{[40]+1} + v^2\,{}_2p_{[40]+1}$$

$$+ v^3\,{}_3p_{[40]+1} + v^4\,{}_4p_{[40]+1}$$

$$= \left(\frac{1}{1.06}\right)\left(\frac{33440}{33485}\right) + \left(\frac{1}{1.06}\right)^2\left(\frac{33378}{33485}\right)$$

$$+ \left(\frac{1}{1.06}\right)^3\left(\frac{33309}{33485}\right) + \left(\frac{1}{1.06}\right)^4\left(\frac{33,231}{33,485}\right)$$

$$= 3.45057$$

(c) $(Ia)_{[40]:\overline{4|}} =$

$$v\,{}_1p_{[40]} + 2v^2\,{}_2p_{[40]}$$

$$+ 3v^3\,{}_3p_{[40]} + 4v^4\,{}_4p_{[40]} =$$

$$\left(\frac{1}{1.06}\right)\left(\frac{33485}{33,519}\right) + 2\left(\frac{1}{1.06}\right)^2\left(\frac{33440}{33519}\right)$$

$$+ 3\left(\frac{1}{1.06}\right)^3\left(\frac{33378}{33,519}\right) + (4)\left(\frac{1}{1.06}\right)^4\left(\frac{33309}{33519}\right)$$

$$= 8.37802$$

(d) $(IA)_{[40]:\overline{4|}} =$

$$v\,\frac{d_{[40]}}{l_{[40]}} + 2v^2\,\frac{d_{[40]+1}}{l_{[40]}} +$$

$\sim [40]$ $\quad\quad\quad\quad$ $l_{[40]}$

$$3v^3 \frac{d_{[40]+2}}{l_{[40]}} + 4v^4 \frac{l_{43}}{l_{[40]}}$$

$$= \left(\frac{1}{1.06}\right)\left(\frac{34}{33519}\right) + 2\left(\frac{1}{1.06}\right)^2\left(\frac{45}{33519}\right)$$

$$+ 3\left(\frac{1}{1.06}\right)^3\left(\frac{62}{33519}\right) + 4\left(\frac{1}{1.06}\right)^4\left(\frac{33378}{33519}\right)$$

$$= 3.16305$$

(e)
$$\left[(1000)^2\left(\frac{{}^2A_{[41]:\overline{4|}} - \left(A_{[41]:\overline{4|}}\right)^2}{d^2}\right)\right]^{\frac{1}{2}}$$

$${}^2A_{[41]:\overline{4|}} = v^2\frac{39}{33467} + v^4\frac{50}{33467}$$

$$+ v^6\frac{69}{33467} + v^8\frac{33309}{33467}$$

$$= 0.62812$$

$$A_{[41]:\overline{4|}} = v\frac{39}{33467} + v^2\frac{50}{33467} +$$

$$v^3\frac{69}{33467} + v^4\frac{33309}{33467}$$

$$= 0.79251$$

$$SD = 1000\sqrt{0.62812 - (0.79251)^2}$$

$$S.D = \frac{1000\sqrt{0.62812-(0.79251)^2}}{\frac{.06}{1.06}}$$

$$= 119.1387$$

(f) $Pr(Y<3) = Pr\left(\frac{1-v^{K+1}}{d} < 3\right)$

$$= Pr\left(1-v^{K+1} < (3)\left(\frac{.06}{1.06}\right) = 0.16981\right)$$

$$= Pr\left(v^{K+1} > 0.83019\right)$$

$$= Pr\left(K+1 < \frac{\ln(0.83019)}{\ln\left(\frac{1}{1.06}\right)}\right)$$

$$= Pr\left(K+1 < 3.19\right)$$

$$= Pr\left(K < 2.19\right) = 1 - {}_3p_x$$

$$= 1 - \frac{l_{44}}{l_{[41]}} = 1 - \frac{33309}{33467} = 0.004721$$

(59) $\ddot{a}_x = a_x + 1 = 10$

$A_x = 1 - d\,\ddot{a}_x$ CLYDE

$0.6 = 1 - d\,(10)$

$d = 0.04$

$1000\bar{A}_x = 1000\left(\frac{i}{\delta}\right)A_x$

$i = .04$

$$1-0.04$$

$$\boxed{\delta = \ln(1+i)}$$

$$= 1000\left[\frac{\frac{.04}{.96}}{\ln\left(1+\frac{.04}{.96}\right)}(0.6)\right]$$

$$= \boxed{612.415}$$

(60) $\ddot{a}_{\overline{x:\overline{n|}}} = 22.9 = \ddot{a}_{\overline{n|}} + {}_{n|}\ddot{a}_x$

$\ddot{a}_{x:\overline{n|}} = 8 = \ddot{a}_x - {}_{n|}\ddot{a}_x = 20 - {}_{n|}\ddot{a}_x$

$\therefore {}_{n|}\ddot{a}_x = 12$

$\therefore \ddot{a}_{\overline{n|}} = 10.90$

Now using BA-II Plus

SET BGN

I/Y = 5
PV = 10.90
PMT = −1
CPT N = $\boxed{115}$

(61)
(a) $100{,}000\,\bar{A}_{85} = 100{,}000\left(\frac{i}{\delta}\right)A_{85}$

$= 100{,}000\,(1.02971)(.73407)$

$= \boxed{75{,}587.92}$

(b) $\bar{A}_{85} = 1 - \delta\,\bar{a}_{85}$

$\bar{a}_{85} = \ddot{a}_{85} - \frac{1}{2} - \frac{1}{12}(\delta + \mu_{85})$

$$\mu_{85} \approx -\frac{1}{2}\left[\ln(1-q_{84}) + \ln(1-q_{85})\right]$$

$$= -\frac{1}{2}\left[\ln(1-.11369) + \ln(1-.12389)\right]$$

$$= 0.126476$$

$\bar{a}_{85} = 4.6980 - \frac{1}{2} - \frac{1}{12}(\ln(1.06) + 0.126476)$

$= 4.182605$

$100{,}000\,\bar{A}_{85} = 100{,}000\left[1 - \ln(1.06)\,(4.182605)\right]$

$= 75{,}628.42$

(62)

Die in first year

$b_1 = 15$ ← first payment

$q_x = 0.05$ ← probability

Die in year 2 pr of 2 payments

$b_2 = 15 + 20/1.06 = 33.868$

$_{1|}q_x = {}_1p_x \, q_{x+1} = (.95)(.1) = .095$

Lives 2 years

$b_3 = 15 + 20/1.06 + \frac{25}{(1.06)^2} = 56.118$

$_2p_x = (.95)(.9) = 0.855$

$E(Y) = 15(.05) + 33.868(.095)$
$+ 56.118(0.855) = 51.946$

$E(Y^2) = (15)^2(.05) + (33.868)^2(.095)$
$+ (56.118)^2(0.855) = 2812.81$

$Var[Y] = E[Y^2] - [E(Y)]^2$

$= 2812.81 - (51.946)^2$

$= 114.42$

63

$E(Y) = 1000\,\ddot{a}_{60} = 11{,}145.40$

$Var(Y) = 1000^2 \; \frac{{}^2A_{60} - (A_{60})^2}{d^2}$

$= (1000)^2 \; \frac{0.17741 - (.36913)^2}{\left(\frac{.06}{1.06}\right)^2}$

$$= (1000)^2(12.84432)$$

$$\sigma = \sqrt{(1000)^2(12.84432)} = 3583.90$$

$$\Pr(Y > 11{,}145.40 + 3583.90)$$

$$= \Pr\left(1000\,\frac{1-v^{K+1}}{d} > 14{,}729.30\right)$$

$$= \Pr\left(\frac{1-v^{K+1}}{d} > 14.73\right)$$

$$= \Pr\left[1-v^{K+1} > (14.73)\left(\frac{.06}{1.06}\right)\right]$$

$$= \Pr\left[v^{K+1} < 1-(14.73)\left(\frac{.06}{1.06}\right)\right]$$

$$= \Pr\left[v^{K+1} < 0.16627\right]$$

$$= \Pr\left[K+1 > \frac{\ln(0.16627)}{\ln\left(\frac{1}{1.06}\right)}\right]$$

$$= \Pr\left[K+1 > 30.79\right]$$

$$= \Pr\left[K > 29.79\right]$$

$$= {}_{30}p_{60} = \frac{l_{90}}{l_{60}} = 0.12927$$

$$= {}_{30}p_{60} = \frac{l_{90}}{l_{60}} = 0.12927$$

64

$$Pr(Y > 12{,}000) =$$

$$Pr\left[100\left(\frac{1-v^{K+1}}{\frac{d^{(12)}}{12}}\right) > 12{,}000\right]$$

where K is measured in months and v is $\left(\frac{1}{1.06}\right)^{\frac{1}{12}}$

$$Pr\left[\frac{1-v^{K+1}}{\frac{d^{(12)}}{12}} > 120\right]$$

$$\boxed{\begin{aligned} 1+i &= \left(1-\frac{d^{(12)}}{12}\right)^{-12} \\ \frac{d^{(12)}}{12} &= 1-(1.06)^{-\frac{1}{12}} = 0.048440 \end{aligned}}$$

$$Pr\left(1-v^{K+1} > 0.58128\right]$$

$$Pr\left[v^{K+1} < 0.41872\right]$$

$$\Pr\left[K+1 > \frac{\ln(0.41872)}{\ln\left[\left(\frac{1}{1.06}\right)^{\frac{1}{12}}\right]}\right]$$

$$= \Pr\left[K+1 > 179.28\right]$$

$$= \Pr\left[K > 178.28\right]$$

= Pr of living 14 YRS 11 months

$$= \frac{l_{54\frac{11}{12}}}{l_{40}} = \frac{\left(\frac{1}{12}\right)(l_{54}) + \left(\frac{11}{12}\right)(l_{55})}{l_{40}}$$

$$= \frac{8,646,841}{9,313,166} = 0.9285$$